- H.w #3 - ENCE 436.
- Mohanad Moayad Shannaff - 1181401.

- Design a square, tied, short column to support axial load $P_u = 390\ t$ in addition to two biaaxial moments $M_{ux} = 95\ t.m$, $M_{uy} = 35\ t.m$.

$f_c' = 35\ MPa$, $\beta_1 = 0.8$

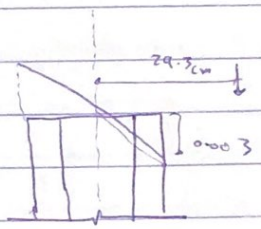
$f_y = 420\ MPa$. , longitudinal bars $= \phi 30$ (along four faces).

$\phi 10$ ties.

$M_{u\,req.} = M_{ux} + 0.55 M_{uz} = 95 + (0.55)(35) = 114.25\ t.m$

$e = 29.3\ cm$. , $\rho_g \approx 2\%$.

$P_{n\,req.} = \frac{390}{0.65} = 600\ t$



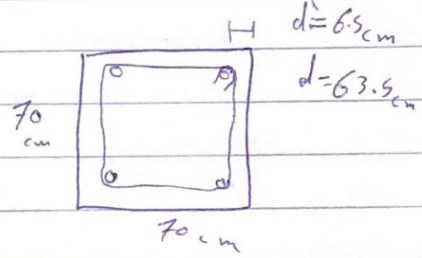
* Preliminary size:

$$A_g \approx \frac{P_n}{0.85 f_c' (0.4)} = \frac{600}{0.85(0.35)(0.4)} = 5042\ cm^2 \approx 70 \times 70\ cm^2$$

$\gamma = 0.814$

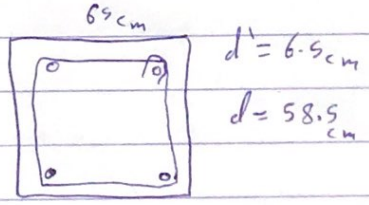
- from chart Columns 3.3.3 - R35-420.8 - Page 81.

$K_n = \frac{P_n}{f_c' A_g} = 0.35$ | $R_n = \frac{P_n e}{f_c' A_g h} = 0.146$



$\Rightarrow \rho = 0.014 = 1.4\%$

take Column $65_{cm} \times 65_{cm} \Rightarrow A_g = 4225\ cm^2$



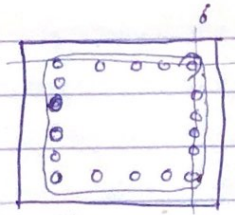
$\gamma = 0.8$

- from chart Column 3.3.3 - R35-420.8 - Page 81.

$K_n = 0.41$, $R_n = 0.183$

$\Rightarrow \rho = 0.027 = 2.7\%$ its good.

$\Rightarrow A_s = 114.075\ cm^2 \Rightarrow 18$ bar $\phi 30$



Longitudinal bar spacing: $S = \frac{65 - 2(4+1) - 6(3)}{5} = 7.4\ cm$

$S_{min} = \max\{4\ cm, 1.5 d_b = 4.5\ cm\} = 4.5\ cm$

$> S_{min}$ ✓

$P_u = 390\ t \quad , M_{ux} = 95\ t.m$

$, M_{uy} = 35\ t.m.$

$e_x = \frac{M_{uy}}{P_u} = 8.97\ cm.$

$e_y = \frac{M_{ux}}{P_u} = 24.36\ cm$

$P_o = 0.85 f_c' A_g + (f_y - 0.85 f_c') A_s$

$= 1753.2\ t$

$X_b = \frac{0.003}{0.0021 + 0.003} (58.5) = 34.4\ cm$

$\epsilon_1 = 0.0024 > \epsilon_y \rightarrow f_{s1} = f_y$

$\epsilon_2 = 0.0013 \rightarrow f_{s2} = 2.6\ t/cm^2$

$\epsilon_3 = 0.00097 \rightarrow f_{s3} = 1.9\ t/cm^2$

$\epsilon_4 = \epsilon_y$

$C_c = 0.85 f_c' (\beta_1 X_b)(65) = 532.2\ t$

$C_{s1} = (6)(7.07)(4.2 - 0.85 f_c') = 165.5\ t$

$C_{s2} = (2)(7.07)(f_{s2} - 0.85 f_c') = 32.6\ t$

$T_3 = (2)(7.07)(f_{s3}) = 26.9\ t$

$T_4 = (6)(7.07) f_y = 178.2\ t$

18Ø30

$f_c' = 35\ MPa$

$f_y = 420\ MPa$

$E_s = 200\,000\ MPa$

$\rightarrow \epsilon_y = 0.0021$

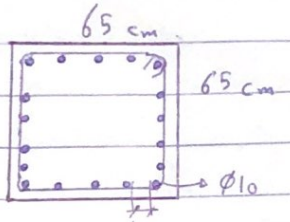
$\rightarrow \beta_1 = 0.8$

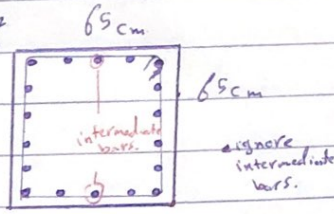
A_s for one bar $= 7.07\ cm^2$

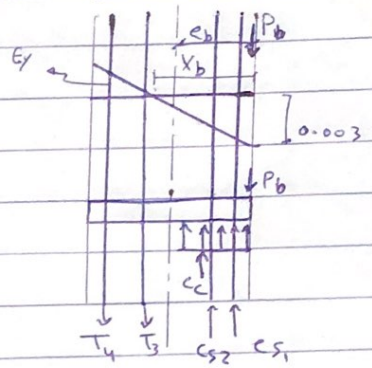
$d' = 6.5\ cm$

$d = 58.5\ cm$

$a = 27.52\ cm$







$+\uparrow \Sigma f_y = 0 \Rightarrow C_c + C_{s1} + C_{s2} - T_3 - T_4 - P_n = 0 \Rightarrow P_b = 525.2\ t$

$\circlearrowleft \Sigma M_{P.C} = 0 \Rightarrow e_b P_b = C_c [32.5 - \frac{a}{2}] + C_{s1}[32.5 - 6.5] + C_{s2}[32.5 - 19.5]$

$+ T_4 [32.5 - 6.5] + T_3 [32.5 - 19.5]$

$e_b P_b = 9973.43 + 8936.2 + 773.5$

$e_b = 37.5\ cm.$

[redacted] , $e_x = 8.97\,cm$ $<$ $e_b = 37.5\,cm$

$P_n = ??$

18 ϕ30

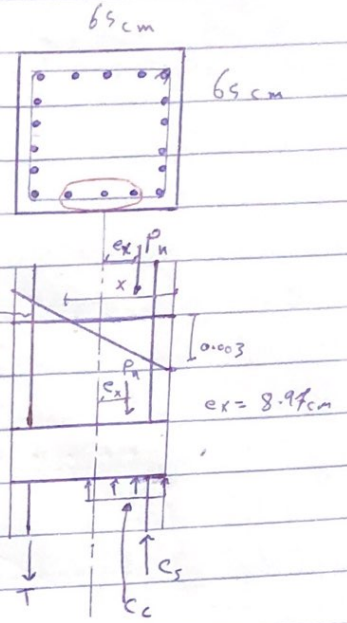
$d' = 6.5\,cm$

$d = 58.5\,cm$

$f_c' = 0.35\ t/cm^2$

$f_y = 4.2\ t/cm^2$

$a = 0.8x$



$\epsilon_s = \frac{0.003}{x}(58.5 - x)$

$\epsilon_s = \frac{0.1755}{x} - 0.003$

$F_s = \frac{351}{x} - 6$

* steel in Compresion will yield.

$C_s = (6)(7.07)(f_y - 0.85 f_c') = 165.5\ t$

$C_c = 0.85 f_c' \cdot a \cdot b = 15.47\,x$

$T = (6)(7.07) F_s = 42.42\left[\frac{351}{x} - 6\right]$

$T = \frac{14889.42}{x} - 254.52$

$\Sigma F_x = 0$ $\quad P_n = C_c + C_s - T = 15.47x + 165.5 + 254.52 - \frac{14889.42}{x}$

$P_n = 15.47\,x + 420.02 - \frac{14889.42}{x}$

$\circlearrowright +$ $\Sigma M_{P.C} = 0 \Rightarrow 8.97[P_n] = C_s[26] + C_c[32.5 - 0.4x] + T[26]$

$138.76\,x + 3767.58 - \frac{133558}{x} = -2314.52 + \frac{387125}{x} + 502.775x - 6.188x^2$

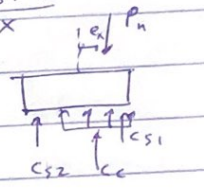
$-6.2x^2 + 364x - 6082.1 + \frac{520683}{x} = 0 \Rightarrow -6.188\,x^3 + 364\,x^2 - 6082.1\,x + 520683 = 0$

[redacted]

$x = 64\,cm \rightarrow$ So all steel in Compresion

$C_{s_1} = 165.5\ t$, $C_{s2} = 6(7.07)\left[6 - \frac{351}{x} - 0.85 f_c'\right] = 242 - \frac{14889.42}{x}$

$C_c = 15.47\,x$, $P_n = 15.47x + \frac{407.5}{} - \frac{14889.42}{x}$

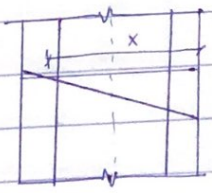


$\circlearrowright +$ $\Sigma M_{P.C} = 0 \Rightarrow 8.97\,P_n + (C_{s2} - C_{s1})26 - C_c(32.5 - 0.4x) = 0$

$138.77\,x + 3655.3 - \frac{133558}{x} + 1989 - \frac{387125}{x} - 502.8\,x + 6.2\,x^2 = 0$

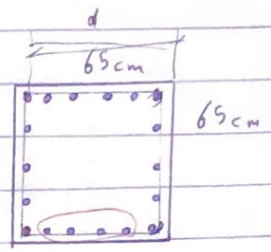
$6.2\,x^2 - 364\,x + 5644.3 - \frac{520683}{x} = 0 \Rightarrow x = 64.8\,cm$

$P_{n,y} = 1180.2\ t.$



$e_y = 24.36$ cm $< e_b = 37.5$ cm

$P_n = ??$



$18\phi 30$

$d' = 6.5$ cm

$d = 58.5$ cm

$f_c' = 0.35$ t/cm²

$f_y = 4.2$ t/cm²

$a = 0.8\,x$

* assume steel in tension not Yeild.

$\epsilon_s = \frac{0.003}{x}(58.5 - x) = \frac{0.1755}{x} - 0.003$

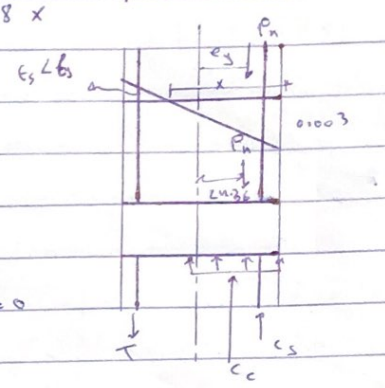
$F_s = \frac{351}{x} - 6$

* steel in Compression will yield.

$T = (5)(7.07)\left(\frac{351}{x} - 6\right) = \frac{12408}{x} - 212.1$

$C_s = (5)(7.07)(f_y - 0.85 f_c') = 137.9$ t

$C_c = 0.85 f_c' a b = 15.47\,x$



$\sum f_y = 0 \Rightarrow P_n = C_c + C_s - T = 15.47x + 350 - \frac{12408}{x}$

$\curvearrowright \sum M_{P_c} = 0 \Rightarrow (24.36)P_n - (T + C_s)(26) - C_c[32.5 - 0.4x] = 0$

$376.85\,x + 8526 - \frac{302259}{x} + 1929.2 - \frac{322608}{x}$

$-502.8\,x + 6.2\,x^2 = 0$

$6.2\,x^2 - 125.95\,x + 10455.2 - \frac{624867}{x} = 0 \Rightarrow 6.2\,x^3 - 125.95\,x^2 + 10455.2\,x - 624867 = 0$

$\Rightarrow x = 40.4$ cm $< d$ The assumption is Correct.

$P_{n,x} = 667.86$ t

Bresler Equation :

$\frac{1}{P_n} \approx \frac{1}{P_c} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_o} = \frac{1}{1180.2} + \frac{1}{667.86} - \frac{1}{1753.2}$

$\frac{1}{P_n} = 1.77 * 10^{-3} \Rightarrow P_n = 563.62$ t $< P_n = 600$ t

When $20\phi 30 \rightarrow P_n = 583.64$ t $P_o = 1815$ t , $P_{nx} = 1180.2$ t, $P_{ny} = 706$ t

$\Rightarrow P_n = 583.64$ t $< P_n = 600$ t

The Column is Not adequate

$\Rightarrow$ Use $22\ \phi 30$

Check spacing:

$S = 5.66\,cm > S_{min.}$

$e_x = 9\,cm$, $e_y = 24.36\,cm$

$P_o = 0.85 f_c' A_g + (f_y - 0.85 f_c') A_s$

$= 1863.5\ t$

22 Ø30

$f_c' = 0.35\ t/cm^2$

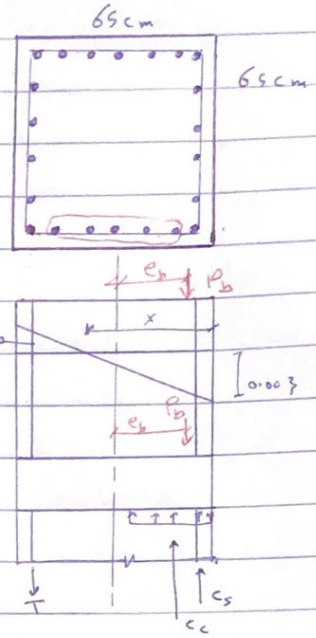
$f_y = 4.2\ t/cm^2$

$d' = 6.5\,cm$

$d = 59.5\,cm.$

$\beta_1 = 0.8$

$a = 0.8x$



balanced case:

$X_b = 34.4\,cm$

$\epsilon_s' = 0.0024 > \epsilon_y$ ∴ Yield.

$T = (6)(7.07) f_y = 178.2\ t$

$C_s = (6)(7.07)(f_y - 0.85 f_c') = 165.5\ t$

$C_c = 0.85 f_c' ab = 532.2\ t$

① $\Sigma f_y = 0 \Rightarrow P_b = C_c + C_s - T = 519.5\ t$

② $\Sigma M_{P.C} = 0 \Rightarrow e_b P_b = (T + C_s)(26) + C_c \left\{32.5 - \frac{a}{2}\right\}$

$e_b = 36.4\,cm.$

when $e_x = 9\,cm < e_b = 36.4\,cm$

assume all steel in Comp.

$\epsilon_{s1} < \epsilon_y$, assume ϵ_{s2} not yield.

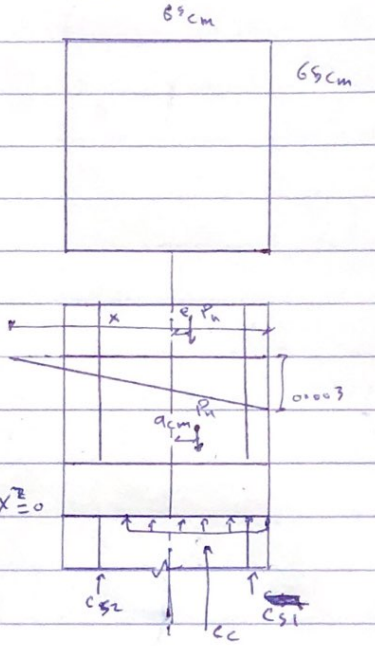
$\epsilon_{s2} = \frac{0.003}{x}(x - 58.5) = 0.003 - \frac{0.1755}{x}$

$C_{s1} = (6)(7.07)(f_y - 0.85 f_c') = 165.5\ t$

$C_c = 15.47\,x$

$C_{s2} = (6)(7.07)\left[6 - \frac{351}{x} - 0.85 f_c'\right] = 241.9 - \frac{14889.4}{x}$

$P_n = 15.47\,x + 407.4 - \frac{14889.4}{x}$



② $\Sigma M_{P.C} = 0 \Rightarrow (9)P_n + (26)C_{s2} - (26)C_{s1} - C_c(32.5 - 9/2) = 0$

$139.23x + 3666.6 - \frac{134005}{x} + 1986.4 - \frac{387124.4}{x} - 502.8x + 6.2x^2 = 0$

$6.2x^3 - 363.57x^2 + 5653x - 521129 = 0$

$X = 64.74\,cm > d$ (The assumption is Correct.)

$P_{n,y} = 1179\ t$

$e_y = 24.36$ cm $< R_b = 36.4$ cm

$P_n = ?$

assume steel in tension not yield.

$\epsilon_s = \frac{0.003}{x}(58.5 - x) = \frac{0.1755}{x} - 0.003$

$F_s = \frac{351}{x} - 6$

∴ steel in compression will yield

$T = (7)(7.07)\left(\frac{351}{x} - 6\right) = \frac{17\,371}{x} - 297$

$C_s = 7(7.07)(4.2 - (0.85)0.35) = 193.1$ t

$C_c = 15.47\,x$

$P_n = 15.47\,x + 490.1 - \frac{17371}{x}$

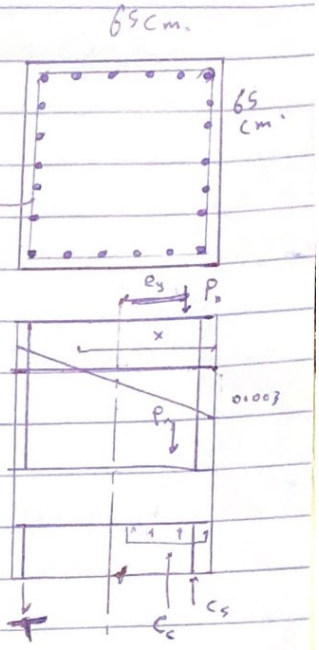
$\curvearrowleft \Sigma M_{P.C} = 0 \Rightarrow (24.36)P_n - (T + C_s)(26) - C_c[32.5 - 0.4x] = 0$

$377\,x + 11\,939 - \frac{423\,158}{x} + 2701 - \frac{451646}{x} - 503x + 6.2x^2 = 0$

$6.2\,x^3 - 126\,x^2 + 14\,640\,x - 874\,804 = 0 \Rightarrow X = 42.64$ cm.

$P_{n_x} = 742.4$ t

22 ϕ30
$f_c' = 35$ MPa
$f_y = 420$ MPa
ϕ10



- Bresler Equation

$\frac{1}{P_n} \approx \frac{1}{P_i} = \frac{1}{P_{n,x}} + \frac{1}{P_{n,y}} - \frac{1}{P_o} = \frac{1}{742.4} + \frac{1}{1179} - \frac{1}{1863.5}$

$P_n = 603$ t $> P_{n_{req.}} = 600$ t

$P_u = \phi P_n = 0.65\,(603)$

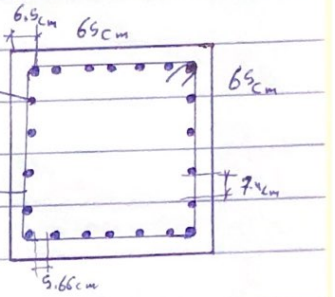
$= 391.95$ t.

Spacing between ties $= S_{max} = \min\{16 d_b, 48 d_t, \text{least dim.}\}$

$S = 48$ cm $= \min\{48_{cm}, 48_{cm}, 65_{cm}\}$

$S \approx 45$ cm

22 ϕ30
$f_c' = 35$ MPa
$f_y = 420$ MPa
ϕ10 @45 cm



Trial Column